In Search of a Contrast Metric: Matching the Perceived Contrast of Gabor Patches at Different Phases and Bandwidths

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The definition of contrast in a complex scene is a long-standing problem. The local contrast in an image may be approximated by the contrast of a Gabor patch of varying phase and bandwidth. Observers' perceived (apparent) contrast, as indicated by matching of such patterns, were compared here to the physical contrast calculated by a number of definitions.

The 2 c/deg 1-octave Gabor patch stimuli of different phases were presented side by side, separated by 4 deg. During each session the subjects (n = 5) were adapted to the average luminance, and four different contrast levels were randomly interleaved. The subject's task was to indicate which of the two patterns was lower in contrast. Equal apparent contrast was determined by fitting a psychometric function to the data. The results of the matching rejected the hypothesis that either the Michelson formula or the King–Smith and Kulikowski contrast metric \( C_{KK} = (L_{\text{max}} - L_{\text{background}})/L_{\text{background}} \) was used by the subjects to set the matching. The use of the Nominal contrast (the Michelson contrast of the underlying sinusoid) as an estimate of apparent contrast could not be rejected.

In a second experiment the apparent contrast of a 1-octave Gabor patch was matched to the apparent contrast of a 2-octave Gabor patch (of Nominal contrast of 0.1, 0.3, 0.6, 0.8) using the method of adjustment. The results of this experiment disagree with the prediction of the Nominal contrast definition as well. The local band-limited contrast measure (Peli, 1990), when used with the modifications suggested by Lubin (1995) as an estimate of apparent contrast, could not be rejected by the results of either experiment. These results suggest that a computational contrast metric based on multi-scale bandpass filtering is a better estimate of apparent perceived contrast than any of the other metrics tested. © 1997 Elsevier Science Ltd

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INTRODUCTION

It is important to be able to measure or calculate the physical contrast in images in a way that is psychophysically valid (i.e., representative of the apparent or perceived contrast). A useful contrast definition or metric should (at least) give equal contrast measures for patterns that are perceived to have equal contrast. This study compared the values calculated by various definitions of physical contrast for patches of gratings (at different phases and bandwidths) when they were perceived to be of equal apparent contrast.

Physical contrast is a property of the stimulus that can be calculated from the luminance distribution in the stimulus. Common to most definitions of contrast is a measure of luminance difference relative to average (or background) luminance. For simple stimuli, such as a light or dark spot or bar on a uniform background \( L_o \), a number of definitions are frequently used. Whittle (1986) pointed out that two patterns that have the same contrast under one such definition will have equal contrast under any other definition that can be shown to be a function solely of the first definition. In particular, he pointed out that the Michelson contrast, \( C_M \), is a function of the Ratio contrast definition, used frequently in the display industry, \( R = L_{\text{max}}/L_{\text{min}} \), where \( L_{\text{max}} \) and \( L_{\text{min}} \) are the maximum and minimum luminance in the pattern, respectively. He defined the contrast \( W = \Delta L/L_{\text{min}} \), where \( \Delta L = L_{\text{max}} - L_{\text{min}} \), which is also a function of \( R \). Thus only one of the three needs to be tested explicitly. On the other hand, the Delta contrast, \( \Delta L/L_o \), is a different function of \( R \) for increments (where \( L_o = L_{\text{min}} \)) and decrements (where \( L_o = L_{\text{max}} \)) stimuli, and as pointed out by Burkhardt et al. (1984) gives a different result as compared to the Michelson metric. Thus, this measure should be evaluated separately from the other three.

Kukkonen et al. (1993) have compared the effects of
the contrast definition used on the shape of the contrast sensitivity functions measured with various stimuli (i.e., gratings, noise, or spots). They found large effects of the metric used on the appearance of the function. They concluded that there may be good reasons for using any of the metrics, but that the Michelson and RMS metrics have intrinsic limitations, especially when contrast is distributed unevenly across the image (as is the case frequently in natural scenes).

Both the RMS and Contrast Energy contrasts they used average the contrast values across the image. Thus, they are reasonable candidates when the average contrast of a uniform texture pattern such as a band limited noise is evaluated. However, in natural scenes the contrast of local features is frequently prominent and for these features a local contrast measure should be applied.

A local band-limited contrast metric, proposed by Peli (1990) and implemented in vision models by others, has the advantage of being applicable to any general complex image. It can be applied locally to determine the contrast of a simple stimulus and may be averaged across the image to provide a global measure as does the RMS measure. This measure (described briefly below) is based on psychophysical notions of the function of the visual system. It calculates the local contrast at various scales (or bands of 1-octave width) and then combines them together. Using quadrature pairs it also explicitly addresses the contrast of a luminance transition (at an edge) and not just increments and decrements as most contrast definitions. Unlike the other measures for which we have analytical expressions, this measure is strictly computational.

As will be shown below, for a localized patch of grating or a local feature in an image, the various formulae or definitions used to calculate the physical contrast may yield widely divergent results even for fairly simple patterns. For a 1-octave Gabor patch, the physical contrast calculated by one definition can be as much as 1.25 times the contrast calculated by another definition. The purpose of this study is to determine what measure of physical contrast corresponds to the apparent contrast perceived by observers viewing a relatively simple quasi-local image—a Gabor patch varying in phase or bandwidth. This type of localized stimulus represents the appearance of local features in complex images, for which we do not have yet an acceptable measure of contrast.

The next section presents the various contrast definitions tested here and their relations to each other, as well as the predictions made by the assumption that observers used such metrics to equate the perceived contrast of two patterns. Experiment 1 compared the Nominal contrast setting selected by observers in a contrast matching of 1-octave patches at various phases. Both the Michelson contrast and the $C_{KK}$ contrast predictions were rejected, but the Nominal contrast prediction could not be rejected. The maximum local contrast calculated using the Peli (1990) measure as modified by Lubin (1995) could not be rejected either. In Experiment 2 we tested for the effect of bandwidth. The contrast of a 1-octave Gabor patch was matched to the contrast of a 2-octave patch. These results rejected the hypothesis that Nominal contrast definition was used by the subject to equate contrasts (the results also reaffirm the rejection of both the Michelson and $C_{KK}$ metrics). The local band-limited contrast measure proposed by Peli (1990), as modified by Lubin (1995) and applied to wide band patterns could not be rejected by the results of either experiment and thus provides a better measurement for local physical contrast in complex patterns in a way that represents the apparent local contrast perceived by observers.

### CONTRAST DEFINITIONS AND PREDICTIONS

If a two dimensional pattern can be represented in the general form:

$$L(x, y) = L_0[1 + N \cdot f(x, y)]$$

where $f(x, y)$ is limited to the range $[-1, 1]$ and $L_0$ is the background luminance, then $N$ is called the Nominal contrast. Note that in the general case, $L_0$ is not equal to $L_{ave}$, the average luminance calculated by integrating $f(x, y)$ over the entire display. $L_0 = L_{ave}$ only when $f(x, y)$ is a zero mean function. For the special case where $f$ is a sinusoidal function, the contrast is called the Michelson contrast (the most widely used definition for physical contrast (Michelson, 1927)) and can be calculated as

$$C_{M} = \frac{L_{max} - L_{min}}{L_{max} + L_{min}}$$

This definition is considered an appropriate measure of apparent contrast of periodic signals that are symmetric in relation to the average luminance, $L_{ave}$, i.e., when

$$L_{max} + L_{min} = 2L_{ave}$$

It is frequently also assumed that the Michelson contrast appropriately describes the apparent contrast seen by observers, even for signals that do not satisfy the requirements stated above. For example, the contrast of a bar on a uniform background frequently has been calculated using the Michelson formula (Whittle, 1986; Burkhart et al., 1984). Similarly, Badecock (1984) proposed ad hoc measures of local contrast for a complex grating pattern composed of the sum of the first and third harmonics. His definitions were equivalent to twice the local Michelson contrast for the cases in which only a local peak and trough were considered. Hess & Pointer (1987) later adopted the same definition for similar patterns.

For the contrast of a localized patch of gratings in a normalized smooth window, another definition has been proposed by King-Smith & Kulikowski (1975) and has been adopted by Wilson (1978) and Swanson et al. (1984). In this formulation, the $C_{KK}$ contrast is defined as:

$$C_{KK} = \frac{L_{max} - L_0}{L_0}$$

The $C_{KK}$ is equal to the Delta contrast for an increment spot.
A third definition for the contrast of complex gratings, which I call the Nominal contrast, \( C_N \), was used by Watson (1987) and was adopted by Peli et al. (1993), among others. According to this definition, the Nominal contrast of the sinusoidal grating in a Gaussian (or any other smooth envelope), is defined as the Michelson contrast of the underlying sinusoid without the envelope. For example, a Gabor patch

\[
L(x, y) = L_e \left[ 1 + C_M \cos(2\pi f_0 x - \varphi) \exp \left( -\frac{x^2 + y^2}{\sigma^2} \right) \right]
\]

(5)

where \( \varphi \) represents the phase of the sinusoid relative to the peak of the Gaussian and \( \sigma \) is a measure of the bandwidth (calculated as full width at 1/e), will have a Nominal contrast \( C_N \). Note that the Nominal contrast differs from the other two definitions in that it cannot be measured in the image, but it is easily determined for these synthetically generated simple patch patterns. This was the main reason for including this definition here.

All three measures of contrast can be applied to a Gabor patch. Such grating patches may differ in the phase, i.e., the relationship of the sinusoid relative to the peak of the Gaussian, and in the bandwidth, which is determined by the width of the Gaussian envelope. All three definitions approach the same value asymptotically when the bandwidth of the patch is decreased (the number of cycles increases). All contrast definitions converge to the same value also when the contrast level is low, thus the definition used matters only when relatively high contrast patterns are used. As noted by Cannon & Fullenkamp (1988), for a cosine phase (\( \varphi = 0 \)) Gabor patch, the Michelson contrast varies with the change in the bandwidth, while the contrast defined by King-Smith and Kulikowski remains unaffected. The Nominal contrast is not affected by either the bandwidth or phase and, therefore, is convenient to use as the basic measure in this study. We have used a cosine phase pattern of 1-octave bandwidth as the test pattern to be matched in perceived contrast to patterns in either a negative cosine phase (\( \varphi = \pi \)) or sine phase (\( \varphi = \pi/2 \)) of the same bandwidth. Examples of the three patterns are presented in Fig. 1 in which all three patterns have the same bandwidth (1-octave) and Nominal contrast (1.0). These patterns differ substantially in their Michelson contrast and King-Smith and Kulikowski contrast. For example, when a sine phase patch has a nominal contrast of 1.0, the Michelson contrast of the sine phase patch is 1.18 times that of the Michelson contrast of the positive cosine phase patch. The \( C_{KK} \) contrast ratio of the two patterns in this case is 0.937.

These ratios are the same for any level of Nominal contrast, since the \( C_{KK} \) is a linear function of \( C_N \). Thus, these values represent the ratios of the Nominal contrasts of a pair of patches which would match in apparent (perceived) contrast if the observer was using the KK definition as his measure. For the Michelson contrast the situation is more complex, since only the Michelson contrast of the sine phase grating is a linear function of \( C_N \). The cosine phase patches' Michelson contrast is not a linear function of \( C_N \) and thus the ratio varies with the Nominal contrast (see the Appendix). The predictions of the Nominal contrast of the test cosine phase pattern calculated to be matched to the negative cosine and sine phase patches using each of the three definitions are given by the curves in Fig. 2(a) and (b), respectively.

Because contrast varies across space in complex images, Peli (1990) has argued that only local contrast is meaningful. To usefully describe image contrast, one must calculate the local physical contrast in an image in a way that will be closely related to the perception of the local contrast by observers. Others have applied the same concept of local band limited contrast with small variations (Menu et al., 1990; Daly, 1992; Lubin, 1995) and found it useful in comparing image quality (Daly, 1992) and other applications (Lubin, 1995). For contrast matching of relatively simple patterns as used here, the maximal local contrast could be equated by the subjects at the equal apparent contrast condition.

The local band-limited definition of contrast proposed by Peli (1990) is a computational rather than an analytical expression. It uses a pyramidal structure of 1-octave wide bandpass filters centered at different scales (frequencies)
Peli (1989) has further suggested that, in addition to the basic bandpass filtering, representing an in-phase mechanism in the visual system, a second set of filtered images representing the quadrature mechanisms (Stromeyer & Klein, 1975) is needed for proper representation of local contrast (Watson, 1987). The response of the in-phase mechanisms, or cells in the visual system, measures only the incremental or decremental changes from local background. The quadrature mechanisms’ response obtained through the Hilbert transform (Papoulis, 1968) of the first set, can be interpreted as measuring transitions from low to high luminance, or vice versa, in a bandlimited signal. A complete description of contrast in complex images should include both the in-phase and the quadrature contrast representations. This quadrature presentation of local contrast was also used by Morrone & Burr (1988), argued for by Daugman (1993), and implemented in the vision models mentioned above (Daly, 1992; Lubin, 1995). Applying the quadrature measure of $C_P$ to the sine phase case (which is higher than the in-phase measure for this pattern) results in a predicted curve running between those of the nominal and Michelson contrasts predictions [Fig. 2(b)]

Combination rules for the in-phase and quadrature contrasts, as well as across scales had to be determined to obtain the desired local measure of contrast. For wider band stimuli, Daly (1992) and Lubin (1995) implemented such models using the Pythagorean sum (a Quick norm with $Q = 2$; Quick, 1974) for the combination rule of in-phase and quadrature phase mechanisms, also termed the contrast energy. Stromeyer & Klein (1975) showed that if these contrast measures are combined using Pythagorean summation, they conveniently describe the physical contrast of a simple sinusoidal grating as uniform everywhere.

Wide band stimuli, such as the 2-octave wide Gabor patch used in Experiment 2, also need to be analyzed across a few bands (or scales) and the results have to be combined. Lubin used a Quick norm with a $Q = 2.4$ as the combination rule where the response, $r$, is:

$$r = \left( \sum_i E_i^Q \right)^{1/Q}$$

where $E_i$ is the contrast energy calculated as the Pythagorean sum of the in-phase and quadrature mechanisms

$$E_i(x, y) = \left[ C_{\text{PL}}^2(x, y) + |H(C_{\text{PL}}(x, y))|^2 \right]^{1/2}$$

where $H(\cdot)$ represents the Hilbert transform.

The maximum of $R(x, y)$, $R(\text{PL})$, is used to derive the predictions tested below.

**EXPERIMENT 1**

**Subjects and methods**

Five adults (20–29 years old) with normal visual acuity participated in the study. Subjects were paid volunteers. All subjects were familiar with the concept of contrast as
it is commonly defined in vision science (graduate students), but naïve to the purpose of the experiment. The subjects compared two patterns presented side by side on a computer display. One pattern was always a positive cosine phase \( (\phi = 0) \) Gabor patch of 1-octave bandwidth; the other was either a negative cosine phase \( (\phi = \pi) \) Gabor patch of the same bandwidth or a sine phase \( (\phi = \pi/2) \) Gabor patch. In all sessions the subject adjusted the contrast of the positive cosine phase test patch.

During each session, four different levels of Nominal contrast \( (0.1, 0.3, 0.6, 0.8) \) of the standard were interleaved. To determine the effects of mean luminance the standard and the test pattern were displayed in one of four different luminance levels. The average luminances of the screen were: 37.5, 3.75, 1.9, 0.75 cd/m². In each presentation, the luminance of the test and the standard were equal, and the subject’s task was to indicate which pattern had the lower contrast. The test pattern Nominal contrast was controlled in 0.02 log unit steps by a PEST routine (Lieberman & Pentland, 1982). Fifty to eighty presentations were used for each level of luminance at each level of contrast to achieve the required level of convergence of the PEST algorithm. Following testing, a psychometric function was fitted to the data, and the threshold for matching apparent contrast for the two patterns was calculated for a 75% correct level. The first 10 responses were not considered in the analysis. The negative cosine phase patch and the positive cosine phase patch were randomly presented on the right or left side of the screen in each trial. The sine phase patch was always presented on the right side of the screen, due to limitations of the system used.

The stimuli were displayed on a 60 Hz noninterlaced, US Pixel monochrome monitor driven by an Adage 3000 image processing system. The screen spans 8 × 8 deg at the viewing distance of 2 m and appeared white. A 10-bit, calibrated look-up table was used to obtain accurate contrast. The 2 c/deg vertical Gabor patches presented side by side were separated by 4 deg. Four minutes of dark adaptation and 3 min of adaptation to the average luminance level preceded the presentation of stimuli. The subject viewed the screen binocularly in a completely dark room with his/her customary distance correction, and was instructed to examine the targets freely using eye movements. The subject indicated which of the two targets had the lower contrast. Following the subject’s response, the patches disappeared and the screen reverted to the average luminance. The subject initiated the next trial when ready. The patterns emerged abruptly and remained on the screen until the subject responded. No feedback was provided.

Results

Averaged results obtained from all five subjects are illustrated in Fig. 2(a,b). Figure 2(a) represents the results for which the standard was a patch at negative cosine phase. Figure 2(b) represents results with the standard being a sine phase patch. The analysis of variance (ANOVA) showed no significant effect of luminance \( (DF = 3, F = 0.53, P = 0.66) \). Therefore, I pooled the ratios measured for all luminance levels. The data points illustrate, for each level, the average Nominal contrast of the test pattern that appeared equal in apparent contrast to the standard. The error bars represent the 95% confidence limits. In both figures the averaged results may be compared with the predictions calculated based on the various contrast definitions (solid lines for the Nominal contrast definition, dotted lines for the \( C_{KK} \) metric, and dashed curves for the Michelson measure). It is clear that the Nominal definition of contrast consistently agrees with the data. The hypothesis that either the Michelson contrast or the King-Smith and Kulikowski measure were used by the subjects is clearly rejected by the data at moderate to high contrast levels.

In testing the prediction of the Peli (1990) definition, it was assumed that the maximum of the local contrast, \( C_p \), was used by the observer to equate the apparent contrast of two patches. The calculated ratios of the maximal contrasts for the 1-octave stimuli of Experiment 1 using the method described in Peli (1990) are given in Fig. 2 as well. Specifically, in this calculation, the low-pass filtered image used for normalization at each scale includes all bands lower than the current scale. For a negative cosine these ratios are very similar to the ratios calculated by the Michelson formula (the two curves cannot be distinguished), and are similarly rejected. For the sine phase case, the predictions (obtained using the quadrature mechanisms’ response) were not as closely matched to the data as for the negative cosine phase. These predictions, however, were closer than those calculated based on the Michelson and KK metrics and could not be statistically rejected. The sine phase results could not be rejected probably because, for the anti-symmetric sine stimulus, the local average luminance is equal to the background luminance. In any case, a rejection in one condition is sufficient.

Lubin (1995) implemented a modified version of the local band-limited contrast definition, \( C_{PL} \), in which the lowpass filtered image used in the normalization is 2 octaves below the current scale, instead of 1 octave below, as in Peli’s implementation (Peli, 1990). Using this modification, the predictions based on \( C_{PL} \) as the metric used by the observer changed as indicated by the curve marked PL in Fig. 2. For negative cosine patch these values are extremely close to those predicted by the Nominal contrast metric. For the sine phase standard patch, these predictions are even closer to the data than those of the nominal contrast metric.

Discussion

The results of Experiment 1 demonstrate that neither the Michelson nor the King-Smith and Kulikowski formulae, when applied to a wide-band Gabor patch at different phases, adequately measures the perceived contrast. In comparison, both the Nominal contrast definition and Lubin’s modification of Peli’s measure cannot be rejected as the possible basis for the subjects matching of the apparent contrast of the various patches.
Further testing is required. If the Nominal contrast is indeed the appropriate measure of physical contrast, then we should find also that a Gabor patch of 1-octave bandwidth is perceived to be equal in contrast to a wider band 2-octave Gabor patch, despite large differences in the Michelson contrast among these signals (Fig. 3) (the $C_{kk}$ and Nominal contrast for these patterns are equal).

Experiment 2 was carried out to test if the apparent contrast of a 2-octave patch is equal to the apparent contrast of a 1-octave patch of the same Nominal contrast. A further difficulty with the Nominal contrast measure is that it gives us no indication how to calculate the relevant physical contrast for any general stimulus other than a Gabor patch. This problem is easily addressed by the Peli–Lubin measure which was also not rejected by the results of Experiment 1.

**EXPERIMENT 2**

Subjects and methods

Five subjects (one of whom had participated in Experiment 1) with normal or corrected to normal vision were tested. They were shown the same type of side-by-side display with the standard always being a positive cosine phase, 2-octave patch of 2 c/deg gratings, and the test stimuli, a positive cosine phase patch of 1 octave at the same spatial frequency. The luminance of the screen in this experiment remained constant at 50 cd/m$^2$, since no effect of luminance was found in Experiment 1. Four levels of standard Nominal contrast were interleaved (0.1, 0.3, 0.6, and 0.8) in each block, once with the standard on the left and once on the right. Five blocks of the same design were administered in the same session.

The stimuli were generated on a Vision Works system using the Image Systems (M21Max) monochrome monitor (yellow in appearance). Using the method of adjustment, the subject changed the test contrast (Nominal) up and down by pressing two buttons on the keyboard (one increasing test contrast and one decreasing) until the apparent contrast was equal to that of the standard patch. The subject was sitting 2 m from the screen in a dark room, and 10 min of adaptation to the average screen luminance preceded the session.

**Results**

Each subject had 10 adjustment results for each contrast level averaged. The averaged responses of the five subjects are illustrated in Fig. 4. The error bars represent the 95% confidence limits. As can be seen, these data clearly reject the Nominal contrast hypothesis. The $C_{kk}$ metric has the same prediction as the Nominal metric for this condition and thus is similarly rejected. Both the $C_{kk}$ and the Michelson metrics were rejected in Experiment 1 and thus need not be retested here. It is nevertheless valuable to see that they fail to predict the results of Experiment 2 as well.

We have tested the hypothesis that the maximum of the contrast measure, $r$ in equation (6), calculated for the stimuli of Experiment 2 using the $Q = 2.4$ suggested by Lubin in combining across bands (scales), could predict the results (using the normalization procedure used by Lubin). As can be seen the data fall clearly on the curve representing these predictions. Changing to $Q = 2.0$ did not change the result. However, $Q = 1$, linear addition, and $Q = 4$ (approximating a winner takes all combination rule) failed to predict all the results. When the combination rule of $Q = 2.4$ was applied to the results of Experiment 1, the results did not change as might be expected, since the stimuli in Experiment 1 were matched in bandwidth to the filters and thus only a small response is recorded in the other bands. Thus, the definition of
contrast proposed by Peli (Peli, 1989, 1990) and as modified by Lubin (1995) could predict all of the results of both experiments described. Note further, that the data of Experiment 1 (sine phase) show an increase in the test-to-standard Nominal contrast ratio as the standard Nominal contrast increases, while the data of Experiment 2 show a decrease in ratio as the Nominal standard contrast is increased. The same trends were noted for the prediction of the Peli–Lubin model.

**GENERAL DISCUSSION**

The formulation of an appropriate definition of contrast is essential for predicting and understanding the perceived contrast of localized image features. The results of both experiments led to the rejection of both the Michelson and the King-Smith and Kulikowski contrast measures, which provide simple convenient analytical expressions for the contrast of simple patterns. It should not be a surprise that these definitions fail to predict the apparent contrast of Gabor patches since they were not designed to do that. The Michelson definition was designed for symmetric patterns only (in fact it was designed for sinusoids only) and thus does not perform for asymmetric patterns such as the cosine phase Gabors. The $C_{KK}$ was designed to apply a measure similar to the Delta contrast to grating patches but it ignores the negative going lobes of such patterns and their effect on apparent contrast. The Nominal contrast measure, which is convenient for use with the Gabor patches, was never intended to be used as a direct measure of contrast for wide-band stimuli.

The operational definition based on the local band-limited computational contrast measure as proposed by Peli (1990) and modified by Lubin (1995) seems to be the only proposal made so far that is not ruled out by experimental results. This measure is based on current understanding of contrast processing in the visual system and it has the added advantage of being applicable to any general complex image. Thus, it is to be expected that it would be better suited to predict contrast matching results. Nevertheless, many more tests are needed before one can conclude that a reliable and veridical measure of contrast in complex images is at hand. We have tested the predictions of the Peli–Lubin measure for the wide-band (positive and negative bars) stimuli tested by Burkhart et al. (1984). Their data (of equal Michelson contrast) fit the prediction up to the Michelson contrast of 0.64 they measured. For a higher level of negative Michelson contrast (0.75), the calculation indicates that a lower positive contrast will be required, resulting in substantial deviation from the Michelson contrast equality found for lower levels of contrast. The local contrast measure can also be averaged across space to yield a measure of contrast similar to the RMS measure, which has been shown to match the perception of noise stimuli (Peli & Brady, 1996).

The local band-limited contrast measure has been implemented successfully in vision models (Daly, 1992; Lubin, 1995) and was used in simulations (Menu et al., 1990; Peli et al., 1991; Peli, 1996). It should be noted that most previous applications of the contrast measure in such models were tested only in threshold context. Even when supra-threshold signals were used, discrimination of just noticeable differences were analyzed and not the apparent contrast perception, as tested in this study. Another important distinction from previous applications is that neither contrast non-linearity nor the masking effects were included or were needed in this case.

**REFERENCES**

Similarly, for the negative cosine phase patch we get

$$C_M(-\cos) = \frac{1.771C_N}{2 - 0.229C_N}.$$  \hspace{1cm} (A2)

The sine phase patch Michelson contrast is linearly related to its Nominal contrast due to symmetry of the pattern around the mean. Thus, for a 1-octave patch

$$C_M(\sin) = 0.937C_N.$$  \hspace{1cm} (A3)

Because of the non-linearity, however, the ratios of $C_M$'s do not accurately represent the ratios of the Nominal contrast of the positive cosine phase test to that of the standard patch when the two have equal Michelson contrast. To find these ratios we have to solve for the $C_N$ ($-\cos$) or the $C_N$ (sin) when the Michelson contrasts are equal, i.e.:

$$C_M(+\cos) = \frac{1.771C_N(+\cos)}{2 + 0.229C_N(+\cos)}$$

$$= \frac{1.771C_N(-\cos)}{2 - 0.229C_N(-\cos)} = C_M(-\cos)$$  \hspace{1cm} (A4)

which when solved yields that under the condition of equal Michelson contrasts the ratios are

$$\frac{C_N(+\cos)}{C_N(-\cos)} = 1.22, 1.161, 1.07, \text{ and } 1.02$$  \hspace{1cm} (A5)

for $C_N(-\cos) = 0.8, 0.6, 0.3, \text{ and } 0.1$, respectively. Solving similarly for the sine phase case leads to:

$$\frac{C_N(+\cos)}{C_N(\sin)} = 1.17, 1.14, 1.10, \text{ and } 1.07.$$  \hspace{1cm} (A6)